Compact lattice U(1) and Seiberg-Witten duality: a quantitative comparison

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based on:

Espriu,Tagliacozzo Phys.Lett. B557 (2003) 125-133

Espriu,Tagliacozzo Phys.Lett. B602 (2004) 137-143

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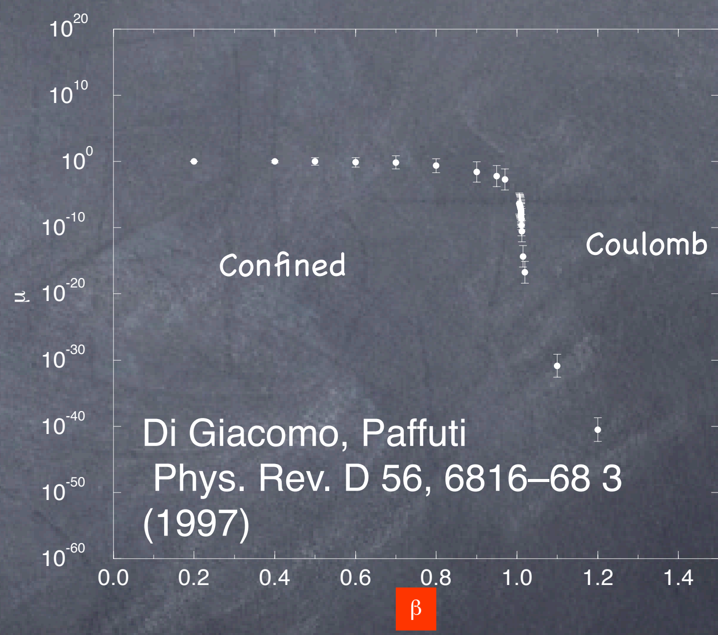

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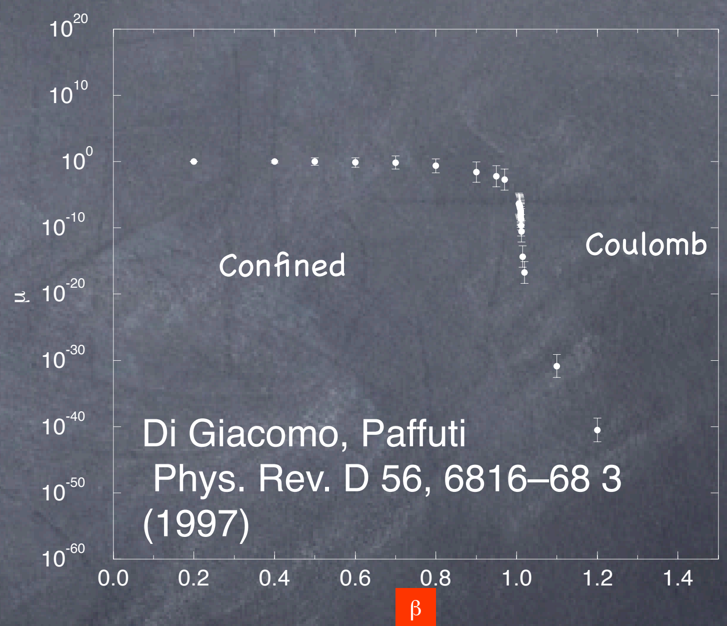

Weak first order transition

Confinement: from Polyakov to S.W.

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G.G. UV d.f

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - a^2\right)^2$$

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Breaking to N=1 in the monopole region

Hierarchies of scales

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Effective FT Picture

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At energies below D_0

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N=0

Lattice

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$$\begin{aligned}
\beta_{g_D} &= \frac{g_D^3}{48\pi^2} \\
\beta_\lambda &= \frac{5\lambda^2}{16\pi^2} - \frac{3\lambda g_D^2}{4\pi^2} + \frac{3g_D^4}{2\pi^2} \\
\gamma_m &= -\frac{3g_D^2}{8\pi^2} \\
\gamma_M &= -2 + \frac{\lambda}{8\pi^2} - \frac{3g_D^2}{8\pi^2}
\end{aligned}$$

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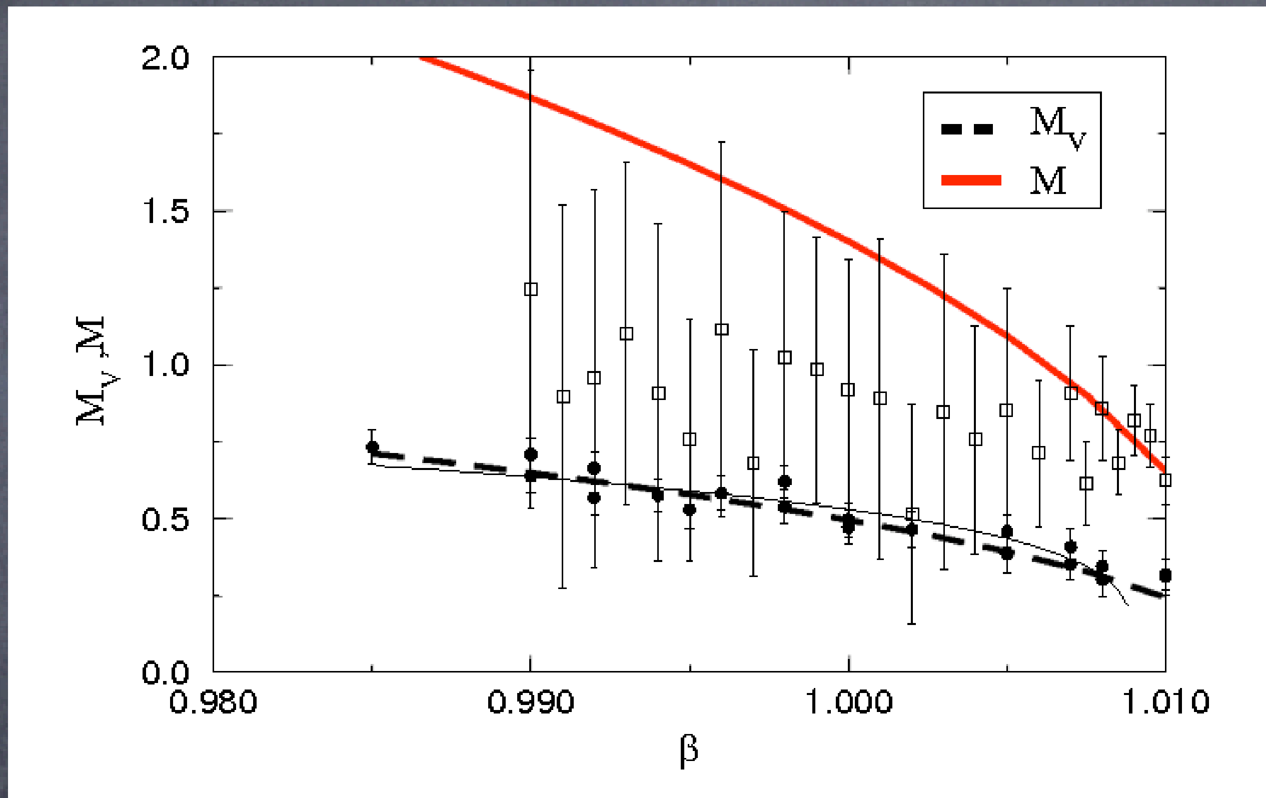

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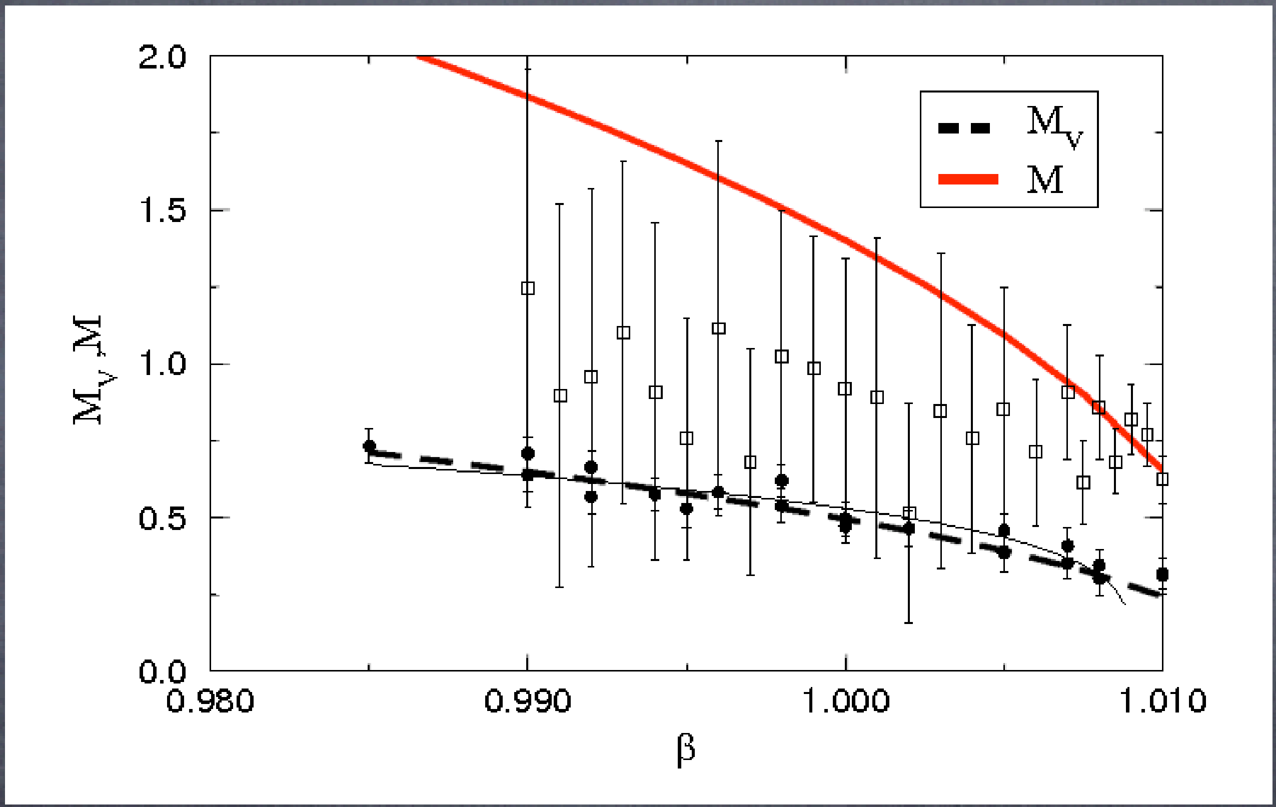

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Type II superconductor

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- Study of other universality classes (3D)